Quantitative Methods in Finance

Step 3: Forecast

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Audience and Prerequisites

Audience:

- These notes were prepared for Kenan Flagler Business School's Daytime MBA program
- The setting is a 14 session course
- These notes serve as reference materials to complement our in-class work using computational tools

Prerequisites:

- Some knowledge of probability, statistics are expected
- Knowledge of finance in general, and asset pricing, in particular is expected

Motivating Case Study

- The motivating case study is portfolio allocation
- However, the concepts and tools are widely applicable to a range of settings within Finance
- We group the concepts into the following functional steps within our case study
  1. Explore - Loading and cleaning data, EDA, etc..
  2. Explain - Factor modeling, etc..
  3. Forecast - Time series models, etc...
  4. Protect - Portfolio allocation, risk measurement, etc..

Housekeeping

- Throughout the slide deck you will see “Q”, which indicates a question to you, the reader
- You will also see “A”, which indicates the associated answer
- It is generally most efficient to learn this material through active participation. Whenever you encounter a “Q”, be sure to try and develop your answer before turning the page to the provided “A” answer
- I recommend reading through these slides before engaging in associated coding exercises

Outline

Task: Forecast Asset Returns

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$E_t[r_{t+h|t}] = ?$
h = forecast horizon

Common Return Forecasting Techniques

- Historical Averages
- Trend Extrapolation
- Time Series
- Factor Models

Forecast Evaluation Measures

$$
\begin{aligned}
r_{t+h|t} &= \beta_1 + \beta_2 X_t + u_{t+h} \\
\hat{r}_{t+h|t} &= \hat{\beta}_1 + \hat{\beta}_2 X_t \\
\hat{u}_{t+h} &= r_{t+h} - \hat{r}_{t+h|t}
\end{aligned}
$$

Forecast Evaluation Measures

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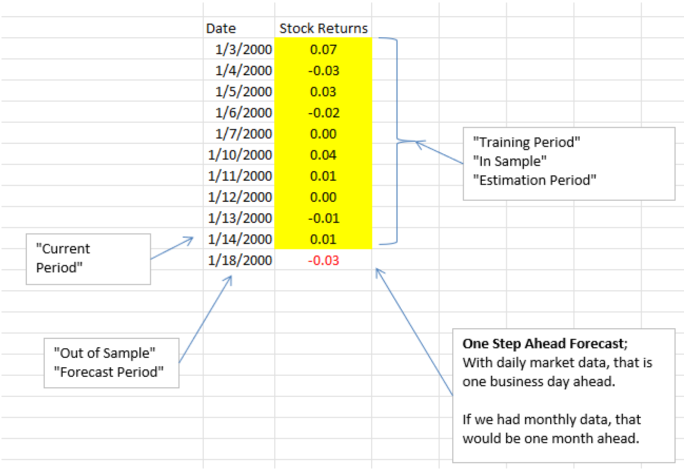
- $ME = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_{t+h|t}$
- $MSE = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_{t+h|t}^2$
- $RMSE = \sqrt{MSE}$
- $MAE = \frac{1}{T} \sum_{t=1}^{T} |\hat{u}_{t+h|t}|$
- $MAPE = \frac{1}{T} \sum_{t=1}^{T} |\frac{\hat{u}_{t+h|t}}{r_{t+h}}|$

Forecast Types

- One Step Ahead vs Multi Step Ahead
- Lagged Regressors vs Time Series Models
- Static vs Dynamic Forecasting
- Backtesting: Training Period, Evaluation Period, Forecast Period

Forecast Types

One Step Ahead Forecasts



Forecast Types

Multi Step Ahead Forecasts

	Daily
Date	Stock Returns
1/3/2000	0.07
1/4/2000	-0.03
1/5/2000	0.03
1/6/2000	-0.02
1/7/2000	0.00
1/10/2000	0.04
1/11/2000	0.01
1/12/2000	0.00
1/13/2000	-0.01
1/14/2000	0.01
1/18/2000	
1/19/2000	
1/20/2000	
1/21/2000	
1/22/2000	0.02

	Weekly
End of Week	Stock Returns
1	0.00
2	0.01
3	0.02

One step ahead weekly forecast

Alter the frequency to match the forecast horizon

Forecast 5 days ahead by converting to weekly data.

Forecast Types

Dynamic Forecasts

Time Series Models
Example: Ret(t) = a + b Ret(t-1) + u

Dynamic Forecasting

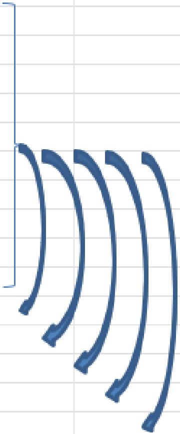
Date	Stock Returns
1/3/2000	0.07
1/4/2000	-0.03
1/5/2000	0.03
1/6/2000	-0.02
1/7/2000	0.00
1/10/2000	0.04
1/11/2000	0.01
1/12/2000	0.00
1/13/2000	-0.01
1/14/2000	0.01
1/18/2000	-0.03
1/19/2000	
1/20/2000	
1/21/2000	
1/22/2000	

Date	Stock Returns
1/3/2000	0.07
1/4/2000	-0.03
1/5/2000	0.03
1/6/2000	-0.02
1/7/2000	0.00
1/10/2000	0.04
1/11/2000	0.01
1/12/2000	0.00
1/13/2000	-0.01
1/14/2000	0.01
1/18/2000	-0.03
1/19/2000	0.03
1/20/2000	
1/21/2000	
1/22/2000	

Date	Stock Returns
1/3/2000	0.07
1/4/2000	-0.03
1/5/2000	0.03
1/6/2000	-0.02
1/7/2000	0.00
1/10/2000	0.04
1/11/2000	0.01
1/12/2000	0.00
1/13/2000	-0.01
1/14/2000	0.01
1/18/2000	-0.03
1/19/2000	0.03
1/20/2000	0.00
1/21/2000	
1/22/2000	

Forecast Types
Static Forecasts

Static Forecasting	
Date	Stock Returns
1/3/2000	0.07
1/4/2000	-0.03
1/5/2000	0.03
1/6/2000	-0.02
1/7/2000	0.00
1/10/2000	0.04
1/11/2000	0.01
1/12/2000	0.00
1/13/2000	-0.01
1/14/2000	0.01
1/18/2000	-0.03
1/19/2000	0.03
1/20/2000	0.00
1/21/2000	0.02
1/22/2000	0.00



Forecast Types

Forecast Errors

			Estimates		
	Date	Stock Returns	Model A	Model B	Model C
	1/3/2000	0.07	0.07	0.07	0.09
	1/4/2000	-0.03	-0.03	-0.03	-0.03
	1/5/2000	0.03	0.03	0.04	0.02
	1/6/2000	-0.02	-0.02	-0.03	-0.01
	1/7/2000	0.00	0.00	0.00	0.01
	1/10/2000	0.04	-0.05	0.08	0.02
	1/11/2000	0.01	0.01	-0.05	-0.02
	1/12/2000	0.00	0.00	0.11	0.03
	1/13/2000	-0.01	-0.01	-0.01	-0.09
	1/14/2000	0.01	0.01	0.14	0.03
	1/18/2000	-0.03	-0.03	-0.03	-0.04
	1/19/2000	0.02	0.03	0.03	0.03
	1/20/2000	0.01	0.00	0.00	0.00
Today	1/21/2000	0.02	0.02	0.02	0.04
	1/22/2000		0.00	0.05	0.04

"Training Period"
"In Sample"
"Estimation Period"

"Evaluation Period"
"Pseudo Out of Sample"

"Out of Sample"
"Forecast Period"

Note: Once the appropriate model is chosen, it is best practice to extend the estimation window to include the evaluation period.

Forecast Types

Model Evaluation

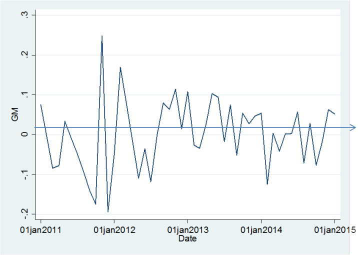
	Errors (Observed-Estimate)				DeMeaned Errors Squared		
Date	Model A	Model B	Model C		Model A	Model B	Model C
1/3/2000	0.00	0.00	0.01		0.0001	0.0006	0.0003
1/4/2000	0.00	0.01	0.01		0.0002	0.0002	0.0001
1/5/2000	0.00	0.00	-0.02		0.0001	0.0004	0.0002
1/6/2000	0.00	-0.01	0.01		0.0000	0.0009	0.0003
1/7/2000	0.00	0.00	0.01		0.0001	0.0003	0.0002
1/10/2000	-0.09	0.04	-0.02		0.0063	0.0003	0.0003
1/11/2000	0.00	-0.06	-0.02		0.0001	0.0065	0.0003
1/12/2000	0.00	0.11	0.02		0.0001	0.0073	0.0008
1/13/2000	0.00	0.00	-0.08		0.0001	0.0005	0.0052
1/14/2000	0.00	0.13	0.02		0.0001	0.0113	0.0008
1/18/2000	0.00	0.00	-0.01		0.0000	0.0007	0.0000
1/19/2000	0.01	0.01	0.01		0.0004	0.0002	0.0001
1/20/2000	-0.01	-0.01	-0.01		0.0000	0.0010	0.0000
1/21/2000	0.00	-0.01	0.02		0.0001	0.0007	0.0005
1/22/2000							
Avg	-0.01	0.02	-0.01	MSE Evaluation	**0.0007**	0.0028	0.0009

Pick the model with the best performance (as measured by lowest MSE) during the evaluation period

Outline

Historical Average

$$E_t[r_{t+h|t}] = \frac{1}{T}\sum_{t=1}^{T} r_{t+h|t}$$



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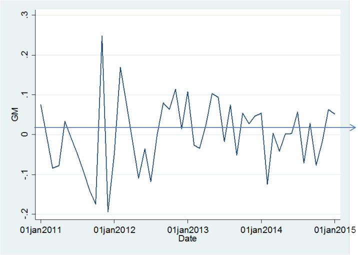

- How far back do you go? — Bigger h requires bigger T

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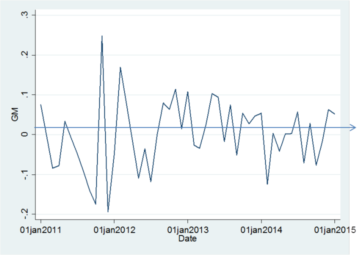
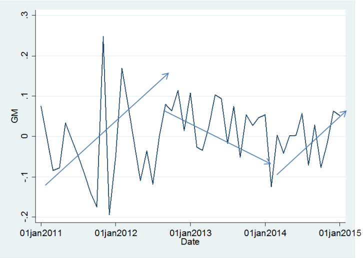


- How far back do you go? — Bigger h requires bigger T
- Caution: Will history repeat itself?

Trend Extrapolation

$$r_{t+h|t} = \alpha + \beta \times t + u_t$$



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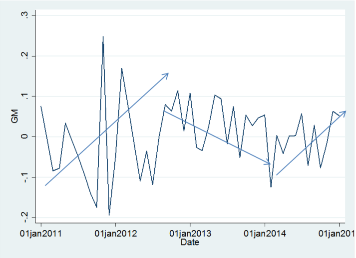

- Over what horizon do you take the trend?

Trend Extrapolation

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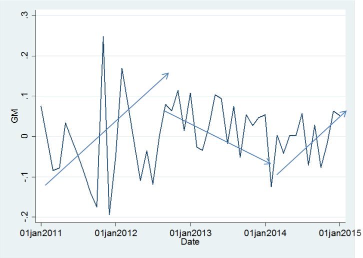

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Outline

Figure: IBM Intraday 2/5/15

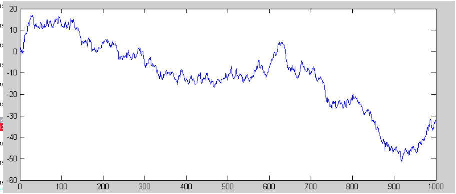

Figure: Simulated Returns

Random Walk

$$r_t = r_{t-1} + \epsilon_t; \quad \epsilon_t \sim N(0,1)$$

Definition

A moving average process of order q [MA(q)] can be represented by

$$r_t = \mu + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j} + \varepsilon_t$$

where $\varepsilon \sim N(0, \sigma^2)$ iid

Definition

An autoregressive process of order p [AR(p)] can be represented by

$$r_t = \sum_{j=1}^{p} \theta_j r_{t-j} + \varepsilon_t$$

where $\varepsilon \sim N(0, \sigma^2)$ iid

Definition

An ARMA(p,q) process can be represented by

$$r_t = \mu + \sum_{j=1}^{p} \theta_j r_{t-j} + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j} + \varepsilon_t$$

Model Choice via Box Jenkins

Definition

The k^{th} autocorrelation is defined as

$$\rho_k = \frac{Cov(y_t, y_{t-k})}{V(y_t)}$$

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Definition

The k^{th} partial autocorrelation is defined as the estimate of θ_k in the AR(k) autoregression. For instance

- $pac_1 = \hat{\theta}_1$ from $y_t = \theta_1 y_{t-1} + \varepsilon_t$
- $pac_2 = \hat{\theta}_2$ from $y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \varepsilon_t$

Model Choice via Box Jenkins

Rules of Thumb

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An AR(p) process is described by

1. An ACF with slow decay
2. A PACF that is (close to) zero for lags larger than p

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2. A PACF with slow decay

Model Choice via Box Jenkins

Rules of Thumb

An AR(p) process is described by

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2. A PACF that is (close to) zero for lags larger than p

An MA(q) process is described by

1. An ACF that is (close to) zero for lags larger than q
2. A PACF with slow decay

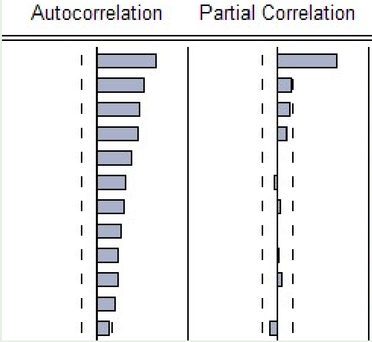
An ARMA process is described by

- Slow decay in both ACF and PACF

Example (Reading the Correlogram)

What model specification seems appropriate given the correlogram below?

Date: 07/11/07 Time: 11:27
Sample: 1996M04 2007M06
Included observations: 135



	AC	PAC	Q-Stat	Prob
1	0.680	0.680	63.865	0.000
2	0.549	0.160	105.71	0.000
3	0.495	0.139	140.06	0.000
4	0.466	0.108	170.72	0.000
5	0.402	0.001	193.72	0.000
6	0.331	-0.031	209.45	0.000
7	0.305	0.034	222.91	0.000
8	0.271	-0.003	233.64	0.000
9	0.248	0.021	242.68	0.000
10	0.239	0.042	251.10	0.000
11	0.213	-0.005	257.89	0.000
12	0.143	-0.096	260.98	0.000

Example (Reading the Correlogram)

The slowly decaying ACF and the spike at 1 lag for the PACF suggest an AR(1) might be appropriate.

Diagnostic Checking

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1. After fitting a potential model, conduct the "usual" overfitting and parameter tests of added variables (t and F tests).

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2. Plot the residuals and look for patterns as a sign of misspecification.
3. Conduct a test of white noise (W.N.) on the residuals
    - H_0 : Residuals are W.N. vs H_a : Residuals are not W.N.
    - Test Statistic: $Q_K = T(T+2)\sum_{k=1}^{K}\frac{1}{T-k}\hat{\rho}_k^2$, where $\hat{\rho}_k$ are the estimated autocorrelation coefficients of residuals $\hat{\varepsilon}_t$ at lag k.
    - Critical Value: $Q_K \sim \chi^2(K)$

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   - Critical Value: $Q_K \sim \chi^2(K)$
4. Search for the smallest AIC and/or BIC values
   - $AIC = \ln(\hat{\sigma}_\epsilon^2) + 2(p+q)/T$
   - $BIC = \ln(\hat{\sigma}_\epsilon^2) + log(T)(p+q)/T$

Outline

1. Anatomy of Forecasts
2. Averages and Time Trends
3. Time Series
4. Factor Models

Factor Models - First Pass

$$r_t - r^f = \alpha + \beta_1 F_{1,t} + \beta_2 F_{2,t} + \cdots + u_t$$

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$$r_t - r^f = \alpha + \beta_1 F_{1,t} + \beta_2 F_{2,t} + \cdots + u_t$$

CAPM	$r_t - r^f = \alpha + \beta[r_t^M - r^f] + u_t$
Fama French	$r_t - r^f = \alpha + \beta_1[r_t^M - r^f] + \beta_2 HML_t + \beta_3 SMB_t + u_t$

Factor Models - First Pass

$$r_t - r^f = \alpha + \beta[r_t^M - r^f] + u_t$$

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Solutions:

- $r_t - r^f = \alpha + \beta[r_{t-1}^M - r^f] + u_t$; Caution: Defies theory.
- Use subjective input for $E_t(r_{t+h}^M)$.
- Forecast $E_t(r_{t+h}^M)$ with a time series (or some other) model.

Factor Models - Second Pass

$$r_i - r^f = \lambda_0 + \lambda_1 \hat{\beta}_i + u_i$$

Forecast returns via: $\hat{\hat{r}}_i - r^f = \hat{\hat{\lambda}}_0 + \hat{\hat{\lambda}}_1 \hat{\beta}_i$, where we use two $(\hat{.})$'s to denote the second pass estimation.

Notice that there is no time series here.

Caution: Assumes β's and λ's are stable over time.